Math 10B with Professor Stankova
Worksheet, Discussion #2; Tuesday, 1/29/2019
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Complementary Counting and PIE

Concepts

1. **Complementary counting** is the problem solving technique (PST) of counting the opposite of what we want and subtracting that from the total number of cases. For instance, instead of counting the number of strings of 6 letters with at least one "E", we would subtract the number of strings without an "E" (the opposite) from total number of 6 letter strings.

   Principle of Inclusion/Exclusion (PIE) is another PST which corrects double counting. It says $|A \cup B| = |A| + |B| - |A \cap B|$ for two sets, $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$. In general, we add the size of all the sets, then subtract all possible pairs of sets, then add back all triplets of sets, subtract all four-sets of sets, etc.

Examples

2. Out of 200 students, there are 100 taking Calculus, 70 taking algebra, and 30 taking both. How many students are taking neither?

3. How many 3 letter sequences do not contain the same letter twice in a row?

Problems

4. True False We can only use the Principle of Inclusion/Exclusion if there are two or three cases or circles.

5. True False Venn diagrams with two circles always look like interlocking rings.

6. Last semester, out of all the students who took both intro chem and 10A, 75% of students passed the intro chem final and 85% passed the 10A final and 70% passed both. There were 50 students who failed both. How many total students took both intro chem and 10A?

7. How many numbers from 1 to 300 are even but not divisible by 3?

8. How many license plates with 3 letters followed by 3 digits have either the 3 letters forming a palindrome or the 3 digits forming a palindrome (or both)?

9. How many numbers less than or equal to 1000 are divisible by 7 or 11 but not both?

10. How many four digit numbers do not have any repeating 1s?

11. How many ways are there to put 7 balls in 3 boxes if each box must have at least one ball?

12. (Challenge) How many ways can we form 2 separate teams out of 5 people if not everyone needs to be on a team but the teams have to have at least one person (e.g. Team A could have person $1, 3$ and team B could have person 5 and person $2, 4$ do not have a team).

Pigeonhole Principle

13. I have 7 pairs of socks in my drawer, one of each color of the rainbow. How many socks do I have to draw out in order to guarantee that I have grabbed at least one pair? What if there are likewise colored pairs of gloves in there and I cannot tell the difference between gloves and socks and I want a matching set?